This thesis has been concerned with a few problems in systems driven at the scale of particles. The problems dealt with here can be extended and elaborated upon in a variety of ways. In 2 we examine the dynamics of a fluid membrane in contact with a fluid containing active particles. In particular, we show that such a membrane generically enters a statistical steady state with wave-like dispersion. While the numerical results are satisfying, a one-step coarse-graining calculation, in line with [66,93], will, we expect, yield a pair of coupled stochastic differential equations (probably KPZ like at least in one dimension) with wave-like dispersion. This calculation in of interest from a theoretical point-of-view. Further, the numerical exploration of the full set of equations is also left for future work, but can be relevant to many biological systems.

In 3 we show that an active fluid confined in an annular channel starts to rotate spontaneously. Further, we predict the existence of banded concentration profile. Such profiles have not yet been observed in experiments. Further, it will be interesting to study what happens to our conclusions if we include the effect of treadmilling in our calculation.

In 4 we describe a solid driven by active particles. Specifically, we only concern ourselves with the polar elastomeric phase of the material. However, the questions regarding the transition into that phase are interesting and have not been explored. How exactly does a polarisation transition happen in an active polar elastomer? Is it the



same as in an active nematic elastomer? What is the nature of the gelation transition in an active polar fluid? What is the dynamics of nematic defects in an elastomer? Can the presence of the elastomer prevent defect separation? We are at present trying to answer these questions.

In 5 we examine the dynamics of an active fluid confined in a channel. It will be interesting to test the prediction about fluctuations in a confined active system, which we show will be normal, in experiments on highly confined actomyosin systems.

In 6 we write down the coupled equations of a conformation tensor and the apolar order parameter. This is a generic framework for studying viscoelastic active fluids. A fuller study of the effect of increasing the cross-linker density in such system remains to be done, both theoretically and experimentally.

In general, we have shown in the thesis that the understanding of active systems can provide a mechanistic explanation of various biological observations. However, at times the comparison between theory and biological experiments become complicated due to the inherently complicated nature of the experimental systems. Thus, for a more rigorous experimental test of the theory, it is necessary to construct cleaner reconstituted systems with possibly as few as three components. Efforts in this direction have recently borne fruit [129]. However, a complete theoretical understanding of the rich behaviour evinced in these systems is as yet lacking. We expect that the conformation tensor theory we developed in chapter 6 will provide an explanation for the anomalous rheological behaviour observed in these systems.

Even in the theoretical front, lot of questions remain to be answered. The dry polar active system, described by the Toner-Tu equations have been shown to undergo a transition to a state with LRO. However, though mean-field theory predicts a second order transition [151, 152, 156], detailed numerical analysis suggests that it is actually first-order with pre-transitional solitonic bands. This has been recently examined by Chate et al. [26] who mapped it to a dynamical system, but a complete theory is still lacking.

Apolar systems present another set of challenges. First, the concentration coupling with the order parameter should create similar pre-transitional effects at the order-disorder transition for this system also. This has been studied to a certain extent [133]. However, the more interesting question concerns the role of defects in apolar systems and whether they allow for the possibility of even QLRO in two dimensions. The +1/2 nematic defect has a polarity, and can thus move balistically [51, 108, 115, 149] in a dry system. However, the −1/2 defect has a three-fold symmetry [27] and its motion is thus purely diffusive. Now consider a pair of +1/2 and −1/2 defect pair that can form due to noise in the system (since it does not violate charge conservation). Depending on the configuration and the kind of activity, this defect pair can unbind at zero temperature. Unbound defects would imply that the order is short-ranged. However, it appears from detailed simulations of an agent based Vicsek-like model of active nematics, that there exists a QLRO nematic in two dimensions [111]! How does an active nematic escape being destroyed by defect unbinding? Does concentration have a major role to play? If so, does making the concentration a non-conserved, and thus fast, variable by, for example, including evaporation-deposition rules in the model studied by Chate et al. [28] destroy the QLRO? Also, does the hydrodynamic theory for Malthusian (i.e. one in which the concentration relaxes fast to a steady value) nematics show only short-ranged order, while the one in which mass is conserved show QLRO? These questions are being studied at present by simulating both the agent-based model due to Chate with evaporation-deposition and the dynamical equation for the active nematic order-parameter. These studies should clarify the role of concentration in assisting apolar order.

It must be borne in mind, however, that numerical simulations of active models are more difficult than their passive counterparts due to the larger number of parameters present in the problem. In passive systems Onsager symmetry relations constrain some parameters. However, the absence of an equivalent rule for systems far away from equilibrium implies that the spatial symmetry allowed couplings will all have independent kinetic coefficients. This increases the size of the parameter space in many problems.

Also, many techniques like Monte Carlo have to be carefully modified to suit such systems.

A new and exciting area of research from the point of view of statistical mechanics of active systems is an examination of collective behaviour of run-and-tumble particles pioneered by Tailleur and Cates [25]. This has led to fruitful active generalisations of models of dynamic critical phenomena like model B and model H. Also, it has led to an exploration of rules for selecting a state in a region of phase coexistence – an out of equilibrium generalisation of the Maxwell construction.

Another interesting avenue is building up active matter equations from microscopics. This has been done for Vicsek model by Thomas Ihle [64,65], for a simple generalisation of Vicsek-type model for both polar and apolar alignment interactions by Bertin et al. and Chate et al. [15, 16, 107], and for a model of hard rods by Marchetti et al. [10, 11]. The issues of closure still remain to be fully resolved however in deriving the macroscopic equations.

A particularly exciting new system that has been recently studied extensively is a collection of chemotactic Janus particles [127]. The far-field interaction in this case does not promote polar order but state with proliferation of asters. The coarse-grained hydrodynamic equations have been derived in this case starting from a microscopic picture of colloids coated axisymetrically with a catalyst in an inhomogeneous concentration of reactants by Saha et al. [127].

Another theoretical issue that plagues the derivation of hydrodynamic equations is that of noise. So far most theories have modelled the noise as Gaussian and white, akin to equilibrium systems, but with unknown strength. However, it is likely that the noise also depends on activity, thus requiring a microscopic picture treating the active forces as stochastic quantities. It is known that multiplicative character of the noise induces interesting features at least in the case of active nematics [104].

Thus, a lot of questions need to be answered if theories of active matter have to graduate from merely offering qualitative explanations of biological experiments to becoming the prototypical theory of systems in which energy input and dissipation both occur at a scale smaller than the coarse-graining volume.